

# Scheda di Allenamento: Disequazioni di 1° grado

## Risoluzione di disequazioni intere

### 1. Disequazioni numeriche intere

Parte Base 1. Risolvi le seguenti disequazioni numeriche intere di primo grado:

1. ■□□  $2x - 3 < x + 5$  [ $x < 8$ ]
2. ■□□  $3(x - 2) \geq 2x - 1$  [ $x \geq 5$ ]
3. ■□□  $4x - 2(x + 3) > 5x - 4$  [ $x < -\frac{2}{3}$ ]
4. ■□□  $\frac{x-1}{2} - \frac{x+1}{3} \leq 1$  [ $x \leq 11$ ]
5. ■□□  $(x - 1)^2 - x(x + 2) < 5$  [ $x > -1$ ]
6. ■□□  $3x - 5 < -2$  [ $x < 1$ ]
7. ■□□  $x - 2 < 7x$  [ $x > -\frac{1}{3}$ ]
8. ■□□  $4x - 3 > 5x + 1$  [ $x < -4$ ]
9. ■□□  $7x - 2 > 3x - 1$  [ $x > \frac{1}{4}$ ]
10. ■□□  $5(x - 1) < 2(x - 3)$  [ $x < -\frac{1}{3}$ ]
11. ■□□  $4[2(1 - x) - 3] > 5x + 1$  [ $x < -\frac{5}{13}$ ]
12. ■□□  $-x - \frac{1}{2} + \frac{x+1}{2} > 0$  [ $x < 0$ ]
13. ■□□  $2(x - 1) + 3(x - 2) < -7$  [ $x < \frac{1}{5}$ ]
14. ■□□  $\frac{1}{2}x - (1 + x) > \frac{3}{2}$  [ $x < -5$ ]
15. ■□□  $4x - 3 < -\frac{2}{3}x + 3$  [ $x < \frac{9}{7}$ ]
16. ■□□  $x - 4(x + 2) \leq 2x - [x - (3 - 4x)]$  [ $\forall x \in \mathbb{R}$ ]
17. ■□□  $x(1 - \frac{1}{3}x) < -\frac{1}{3}x^2 + 2$  [ $x < 2$ ]
18. ■□□  $6x + 7 > \frac{1}{3}(9x - 3)$  [ $x > -\frac{8}{3}$ ]
19. ■□□  $\frac{3}{2}(x + \frac{1}{2}) > 2(x + \frac{1}{2}) - \frac{1}{2}(x - \frac{1}{2})$  [impossibile]
20. ■□□  $\frac{x-2}{3} + \frac{1}{2} < \frac{x+1}{6}$  [ $x < 2$ ]
21. ■□□  $5x - 2 < 3x + 4$  [ $x < 3$ ]
22. ■□□  $2(x - 3) \geq 4x - 10$  [ $x \leq 2$ ]

23. ■□□  $\frac{x}{3} - 1 > \frac{x}{2}$  [ $x < -6$ ]
24. ■□□  $3x + 5 \leq 3(x + 2) - 2$  [impossibile]
25. ■□□  $\frac{x-1}{2} + \frac{x}{4} < \frac{2x+1}{4}$  [ $x < 3$ ]
26. ■□□  $4 - 2x > 6(1 - x)$  [ $x > \frac{1}{2}$ ]
27. ■□□  $x(x + 1) - x^2 < 3x - 4$  [ $x > 2$ ]
28. ■□□  $\frac{1}{3}(2x - 1) \geq \frac{1}{2}(x + 1)$  [ $x \geq 5$ ]
29. ■□□  $(x - 1)^2 \leq x(x - 3) + 5$  [ $x \leq 4$ ]
30. ■□□  $2x - \frac{x+2}{5} > \frac{3x-1}{2}$  [ $x > -\frac{1}{3}$ ]

## 2. Diseguazioni numeriche avanzate

**Parte Avanzata 1.** Risolvi le seguenti disequazioni numeriche intere di livello avanzato, prestando attenzione allo sviluppo dei prodotti notevoli:

31. ■■■■  $x^2(2 - x) + (x - 2)^3 \leq -5(1 - 2x) + (2x + 1)(1 - 2x)$  [ $x \leq 2$ ]
32. ■■■■  $(\frac{1}{2} - x)^2 - (x + 1)^2 < -[1 - (\frac{2x+1}{6})] + \frac{1+2x}{3}$  [ $x > -\frac{1}{16}$ ]
33. ■■■■  $(\frac{1}{3} - \frac{5}{3}x)^2 < \frac{5}{9}x(x - 2) + (x - \frac{4}{9}x)4x$  [impossibile]
34. ■■■■  $\frac{(x-1)(x+1)}{2} + [x - \frac{5}{4} - (\frac{1}{2}x + 1)] - \frac{(x-3)^2}{2} > 0$  [ $x > \frac{29}{11}$ ]
35. ■■■■  $\frac{1}{2}(x - \frac{4}{3}) + (3 - \frac{x}{2})^2 < \frac{x^2}{4} + \frac{x-1}{3} - \frac{2x+5}{2} + 2$  [ $x > 5$ ]
36. ■■■■  $\frac{4}{9}[x + \frac{3(x-1)}{4}] + (x + \frac{1}{3})^2 \geq \frac{1}{3} + (x - \frac{1}{3})(x + 2)$  [ $x \leq \frac{1}{2}$ ]
37. ■■■■  $2(2x - 1)(2x + 1) - 6(x - 2)^2 \leq (x - 3)^2 - [3(3 + x)(3 - x) - 2(x + 1)(1 - x)]$   
[ $x \leq \frac{1}{3}$ ]
38. ■■■■  $3(x - \frac{1}{3})(x + 2) + (5x - 1)^2 < 9x(\frac{1}{3}x - 1) + 40 + (5x + 1)^2$  [ $x > -7$ ]
39. ■■■■  $\frac{1}{3}(9 - \frac{3x}{2}) - \frac{2}{5}(\frac{15+5x}{2}) > (3 - 2x)^2 - (2x + 1)(2x - 1) - 6$  [ $x > \frac{8}{21}$ ]
40. ■■■■  $(x + 3)^3 - 4[x + 5 - (x + 8)]^3 > (x - 3)^2(x + 3) + 12(x + 1)^2$  [ $x > -8$ ]
41. ■■■■  $(x - 1)^3 - (x + 1)^3 > 2x - 2 - 6x^2 + 2(x + 1)(x - 1) - 2(x - 2)^2$  [ $x < 1$ ]
42. ■■■■  $\frac{1-x}{1-\frac{1}{3}} + \frac{1-\frac{2x}{3}}{1+\frac{1}{3}} > 7x + (1 - x)^2 - (x + 1)(x - 1)$  [ $x < \frac{1}{28}$ ]
43. ■■■■  $(\frac{1}{2} - x)^2 - \frac{1-2x}{3} \leq (1 + x)^2 - (1 - \frac{3x-1}{6})$  [ $x \geq \frac{1}{34}$ ]
44. ■■■■  $2(x - \frac{1}{2})(x + \frac{1}{4}) - x(x + 2) \leq (x - \frac{1}{2})^2 - \frac{1}{2}$  [ $x \geq 0$ ]
45. ■■■■  $4(5x - 1) + 2(3x + 1)^2 > 3x(6x + 5) - 2x - 3$  [ $x > -\frac{1}{19}$ ]

46. ■■■■  $(x+2)^3 - (x-1)^3 < 9x(x+2) + 5$   $[x > \frac{4}{9}]$
47. ■■■■  $\frac{(2x-1)^2}{2} - 2x(x+1) \leq \frac{1-8x}{2}$   $[\forall x \in \mathbb{R}]$
48. ■■■■  $\frac{(x-2)(x+2)}{3} - \frac{(x-1)^2}{3} < \frac{x+5}{2}$   $[x < 25]$
49. ■■■■  $(x + \frac{1}{2})^2 - (x - \frac{1}{2})^2 \geq 2x - \frac{x-1}{4}$   $[x \geq 1]$
50. ■■■■  $(x-2)^3 - x(x-3)^2 < 5x - 12$   $[x > 2]$
51. ■■■■  $\frac{(x+2)^2}{2} - \frac{x^2+4x}{4} \leq (\frac{x-2}{2})^2 + 3x$   $[x \geq 1]$
52. ■■■■  $6x : (-\frac{3}{2}) - (\frac{4}{5}x) : (-\frac{2}{5}x) - 5^0 < -\frac{x}{3}$   $[x > \frac{3}{11}]$
53. ■■■■  $(-3)^5 : (3^2)^2x + (-2^2)^3 : (-2)^4 \leq (-2)^3 \cdot (-1)^5x$   $[x \geq -\frac{4}{11}]$
54. ■■■■  $(\frac{1}{3} - \frac{1}{2})^{-1}x - (-\frac{1}{2})^{-2} \geq 2x$   $[x \leq -\frac{1}{2}]$
55. ■■■■  $(2 - \frac{1}{2})^{-2}x - (1 - \frac{1}{3})^{-1} \geq \frac{4}{9}x - 2$   $[\forall x \in \mathbb{R}]$
56. ■■■■  $\frac{(x-1)^3}{3} - \frac{x^2(x-3)}{3} \leq x - \frac{2x-1}{6}$   $[x \leq \frac{3}{2}]$
57. ■■■■  $(\frac{1}{2})^{-2}(x-1) - (-\frac{2}{3})^{-1} \cdot (\frac{2}{3})x > (\frac{1}{3})^{-1}(x+2)$   $[x > 5]$
58. ■■■■  $(2x-1)^2 - (x+1)^3 + x(x-1)^2 \geq -x^2 - 4$   $[x \leq \frac{2}{3}]$
59. ■■■■  $2[x - \frac{1}{2}(x - \frac{1}{3})] - \frac{x+1}{3} < \frac{3x-1}{2}$   $[x > \frac{3}{5}]$
60. ■■■■  $\frac{(x-2)^2}{3} - \frac{x(x-4)}{3} \leq \frac{1}{2} - \frac{2x-3}{4}$   $[x \leq -\frac{1}{6}]$
61. ■■■■  $[\frac{(x-2)^2}{2} - \frac{(x+2)^2}{2}]^2 - 16x^2 > (x-1)^3 - x^2(x-3) + 1$   $[x < -1]$
62. ■■■■  $\frac{1}{2}[(x - \frac{1}{2})^2 - (x + \frac{1}{2})^2] \geq \frac{x-1}{3} - \frac{1}{2}(x - \frac{x-1}{2})$   $[x \leq \frac{1}{9}]$
63. ■■■■  $(\frac{1}{3} - 1)^{-2} \cdot (x - 3) - (1 - \frac{2}{3})^{-1} \cdot x \leq (\frac{x-1}{2}) \cdot (-2)^2$   $[x \geq -\frac{19}{17}]$
64. ■■■■  $\frac{(x+1)^3 - (x-1)^3}{6} - \frac{(2x+1)^2}{4} < \frac{x-1}{2} - x(x+1)$   $[x < 3]$
65. ■■■■  $2x - \{\frac{x-1}{2} - [x - \frac{1}{3}(x+1)]\} \geq \frac{(x-1)(x+1)}{2} - \frac{x^2-3}{2}$   $[x \geq -\frac{1}{13}]$
66. ■■■■  $[\frac{(2x-1)^2}{4} - \frac{(2x+1)^2}{4}]^2 - 2x^2 > (x+1)^3 - x(x+2)^2 + 5$   $[x < -3]$
67. ■■■■  $\frac{1}{3}[(x - \frac{2}{3})^2 - (x + \frac{1}{3})^2] \geq \frac{1-2x}{9} - \frac{1}{3}(x - \frac{x+2}{3})$   $[x \geq \frac{1}{2}]$
68. ■■■■  $(1 - \frac{3}{4})^{-2} \cdot \frac{x-1}{8} - (2 - \frac{1}{2})^{-1} \cdot 3x \leq \frac{(x-2)^2}{2} - \frac{x^2-4}{2}$   $[x \geq 1]$
69. ■■■■  $\frac{(x+2)^3 - (x-2)^3}{12} - \frac{(x-1)^2}{2} < \frac{2x+3}{3} - \frac{1}{2}(x^2 - 4x)$   $[x < \frac{1}{6}]$
70. ■■■■  $3x - \{\frac{2x-1}{3} - [x - \frac{1}{2}(x-3)]\} \geq \frac{(x+2)(x-2)}{2} - \frac{x^2+2x}{2}$   $[x \geq -\frac{11}{17}]$

### 3. Disequazioni letterali

**Parte Avanzata 2.** Risolvi le seguenti disequazioni letterali intere nell'incognita  $x$  al variare del parametro in  $\mathbb{R}$ :

71. ■■■□  $(a - 2)x < 3$   $[a < 2, x > \frac{3}{a-2}; a = 2, \forall x \in \mathbb{R}; a > 2, x < \frac{3}{a-2}]$

72. ■■■□  $3ax - a - 6a > -2 + 4ax$   $[a > 0, x < \frac{2-7a}{a}; a < 0, x > \frac{2-7a}{a}; a = 0, \forall x \in \mathbb{R}]$

73. ■■■□  $a(x - 1) > 1 + 2x$   $[a > 2, x > \frac{a+1}{a-2}; a < 2, x < \frac{a+1}{a-2}; a = 2, \text{impossibile}]$

74. ■■■□  $ax - a^2 \geq 3x - 9$   $[a > 3, x \geq a + 3; a < 3, x \leq a + 3; a = 3, \forall x \in \mathbb{R}]$

75. ■■■□  $4x - a \leq a(x + 2)$   $[a < 4, x \leq \frac{3a}{4-a}; a = 4, \forall x \in \mathbb{R}; a > 4, x \geq \frac{3a}{4-a}]$

76. ■■■□  $(a - 1)x - (1 - a)(1 + a) < 0$   
 $[a > 1, x < -(a + 1); a = 1, \nexists x \in \mathbb{R}; a < 1, x > -(a + 1)]$

77. ■■■□  $(1-a)(x+1) \leq x-2+a(x-1)$   $[a > 0, x \geq \frac{3}{2a}; a = 0, \nexists x \in \mathbb{R}; a < 0, x \leq \frac{3}{2a}]$

78. ■■■□  $a(x - 3) < x - 1$   $[a > 1, x < \frac{3a-1}{a-1}; a = 1, \forall x \in \mathbb{R}; a < 1, x > \frac{3a-1}{a-1}]$

79. ■■■□  $b(2x - 1) \geq -2x + 4 + b$   $[b > -1, x \geq \frac{b+2}{b+1}; b = -1, \nexists x \in \mathbb{R}; b < -1, x \leq \frac{b+2}{b+1}]$

80. ■■■□  $3x - a(x - a) < (a - 1)(a + 1) + a + 4$   
 $[a > 3, x > \frac{a+3}{3-a}; a < 3, x < \frac{a+3}{3-a}; a = 3, \forall x \in \mathbb{R}]$

81. ■■■□  $x - a(x - 1) > x(1 + a)$   $[a < 0, x > \frac{1}{2}; a = 0, \nexists x \in \mathbb{R}; a > 0, x < \frac{1}{2}]$

82. ■■■□  $x(1 + bx) + x(x - b) < (x^2 + 1)(1 + b)$   
 $[b < 1, x < \frac{1+b}{1-b}; b > 1, x > \frac{1+b}{1-b}; b = 1, \forall x \in \mathbb{R}]$

83. ■■■□  $(x + a)^3 - 4ax(a - x) > 2x(x + a)^2 + (a - x)^3$  [impossibile]

84. ■■■□  $(5x - a)(x - 2a) + (a + x)^2 \leq (x - a)(x + a) + (x - a)(5x + 4a)$   
 $[a < 0, x \leq a; a > 0, x \geq a; a = 0, \forall x \in \mathbb{R}]$

85. ■■■□  $bx^2 + 4 + x(2b - 1) > b(2 + x)(b + x) - 2b(b + 1)$   $[x < \frac{2b+4}{b^2+1}]$

86. ■■■□  $x - a(x - 1) > x(1 + a)$   $[a < 0, x > \frac{1}{2}; a = 0, \nexists x \in \mathbb{R}; a > 0, x < \frac{1}{2}]$

87. ■■■□  $x(1 + bx) + x(x - b) < (x^2 + 1)(1 + b)$   
 $[b < 1, x < \frac{1+b}{1-b}; b > 1, x > \frac{1+b}{1-b}; b = 1, \forall x \in \mathbb{R}]$

88. ■■■□  $(x + a)^3 - 4ax(a - x) > 2x(x + a)^2 + (a - x)^3$  [impossibile]

89. ■■■□  $(5x - a)(x - 2a) + (a + x)^2 \leq (x - a)(x + a) + (x - a)(5x + 4a)$   
 $[a < 0, x \leq a; a > 0, x \geq a; a = 0, \forall x \in \mathbb{R}]$

90. ■■■□  $bx^2 + 4 + x(2b - 1) > b(2 + x)(b + x) - 2b(b + 1)$   $[x < \frac{2b+4}{b^2+1}]$

91. ■■■□  $\frac{x-2}{a-4} > 0$   $[a < 4, x < 2; a = 4, \text{senza significato}; a > 4, x > 2]$

92. ■■■■  $\frac{bx-2}{b} \geq 1 + \frac{1}{b}x$   
[ $b = 1$ , imp.;  $b = 0$ , perde di significato;  $b < 0 \vee b > 1$ ,  $x \geq \frac{b+2}{b-1}$ ;  $0 < b < 1$ ,  $x \leq \frac{b+2}{b-1}$ ]
93. ■■■■  $\frac{a-2x}{2a} - \frac{1}{a} \leq 0$  [  $a < 0$ ,  $x \leq \frac{a-2}{2}$ ;  $a = 0$ , senza sign.;  $a > 0$ ,  $x \geq \frac{a-2}{2}$  ]
94. ■■■■  $\frac{(b-2)x+b}{b} \geq 0$   
[  $b < 0 \vee b > 2$ ,  $x \geq -\frac{b}{b-2}$ ;  $b = 0$ , senza sign.;  $0 < b < 2$ ,  $x \leq -\frac{b}{b-2}$ ;  $b = 2$ ,  $\forall x \in \mathbb{R}$  ]
95. ■■■■  $x + \frac{1}{2a} - \frac{1}{2} < \frac{x}{a}$   
[  $a < 0 \vee a > 1$ ,  $x < \frac{1}{2}$ ;  $a = 0$ , senza sign.;  $0 < a < 1$ ,  $x > \frac{1}{2}$ ;  $a = 1$ , imp. ]