

# Scheda di Allenamento: Sistemi Lineari

## esercizi sulle tecniche di risoluzione

### 1. Sistemi lineari: Metodo di sostituzione

**Parte Base 1.** Risolvi i seguenti sistemi lineari utilizzando esclusivamente il metodo di sostituzione:

$$1. \begin{cases} x + y = 3 \\ x - y = 1 \end{cases} \quad (2, 1) \quad 12. \begin{cases} -x + 2y = 4 \\ 5x - y = -11 \end{cases} \quad (-2, 1)$$

$$2. \begin{cases} 2x + y = 5 \\ x - y = 1 \end{cases} \quad (2, 1) \quad 13. \begin{cases} x + y = 7 \\ 3x - 2y = 6 \end{cases} \quad (4, 3)$$

$$3. \begin{cases} x = 2y + 1 \\ 3x - y = 8 \end{cases} \quad (3, 1) \quad 14. \begin{cases} 2x - y = 1 \\ x + 2y = 8 \end{cases} \quad (2, 3)$$

$$4. \begin{cases} y = x - 4 \\ 2x + y = 5 \end{cases} \quad (3, -1) \quad 15. \begin{cases} y = -2x + 5 \\ 3x + 4y = 15 \end{cases} \quad (1, 3)$$

$$5. \begin{cases} 3x + 2y = 7 \\ x - y = 4 \end{cases} \quad (3, -1) \quad 16. \begin{cases} x - y = -2 \\ 4x + 3y = 27 \end{cases} \quad (3, 5)$$

$$6. \begin{cases} 4x - 3y = 10 \\ y = 2x - 6 \end{cases} \quad (4, 2) \quad 17. \begin{cases} 6x - y = 11 \\ 2x + y = 5 \end{cases} \quad (2, 1)$$

$$7. \begin{cases} x + 3y = 9 \\ 2x - y = 4 \end{cases} \quad (3, 2) \quad 18. \begin{cases} 3x - 4y = -6 \\ x + 2y = 8 \end{cases} \quad (2, 3)$$

$$8. \begin{cases} 5x - 2y = 4 \\ x + y = 5 \end{cases} \quad (2, 3) \quad 19. \begin{cases} y = x + 2 \\ 5x - 3y = 4 \end{cases} \quad (5, 7)$$

$$9. \begin{cases} y = 3x - 1 \\ 4x + 2y = 18 \end{cases} \quad (2, 5) \quad 20. \begin{cases} 2x + 5y = -4 \\ x - 3y = 9 \end{cases} \quad (3, -2)$$

$$10. \begin{cases} x - 4y = -10 \\ 3x + y = 9 \end{cases} \quad (2, 3) \quad 21. \begin{cases} \frac{x+y}{3} = \frac{x}{2} + 1 \\ x - \frac{y-2}{2} = \frac{x+y}{6} \end{cases} \quad (2, 4)$$

$$11. \begin{cases} 2x + 3y = 12 \\ x = 6 - y \end{cases} \quad (6, 0) \quad 22. \begin{cases} \frac{2x-y}{7} = x + y - 1 \\ \frac{x}{3} + y = 0 \end{cases} \quad (3, -1)$$

$$\begin{array}{ll}
23. \begin{cases} \frac{x+3y}{4} = 1 \\ \frac{x}{2} - \frac{y}{3} = \frac{x-y}{4} - \frac{2}{3} \end{cases} & (-2, 2) & 32. \begin{cases} \frac{x+y}{4} = \frac{y-2}{2} \\ x + \frac{y}{2} = 2 \end{cases} & (0, 4) \\
24. \begin{cases} x - \frac{y+2}{3} = 3 \\ \frac{2x-y}{7} = y \end{cases} & (4, 1) & 33. \begin{cases} \frac{x-y}{2} + \frac{x+y}{3} = 5 \\ \frac{x-1}{2} = y - 2 \end{cases} & (7, 5) \\
25. \begin{cases} \frac{x+y}{2} = x + 2 \\ \frac{y-x}{4} = \frac{y}{5} \end{cases} & (1, 5) & 34. \begin{cases} \frac{x+1}{2} = y \\ \frac{x-y}{2} + y = -2 \end{cases} & (-3, -1) \\
26. \begin{cases} \frac{x-2}{4} + y = 1 \\ \frac{x+y}{3} = \frac{x}{2} - 1 \end{cases} & (6, 0) & 35. \begin{cases} \frac{2x+y}{3} + \frac{x-y}{2} = 1 \\ x - \frac{y}{2} = \frac{1}{2} \end{cases} & (1, 1) \\
27. \begin{cases} \frac{2x+y}{5} = -1 \\ x - \frac{y}{3} = 0 \end{cases} & (-1, -3) & 36. \begin{cases} \frac{x}{2} + \frac{y}{2} = 1 \\ \frac{x-y}{3} = x + y \end{cases} & (4, -2) \\
28. \begin{cases} \frac{x-y}{3} = \frac{x+1}{6} \\ \frac{y+2}{4} = \frac{x-3}{2} \end{cases} & (5, 2) & 37. \begin{cases} \frac{x+y}{2} = -1 \\ \frac{x+2}{3} = -y + 2 \end{cases} & (-5, 3) \\
29. \begin{cases} \frac{x}{2} + 2y = 0 \\ \frac{x+y+3}{5} = \frac{x}{4} + 1 \end{cases} & (-4, 1) & 38. \begin{cases} \frac{y}{3} - \frac{x}{2} = 1 \\ \frac{x+y}{4} = x \end{cases} & (2, 6) \\
30. \begin{cases} \frac{x+y}{2} - \frac{x-y}{3} = 3 \\ \frac{2x}{3} = y - 1 \end{cases} & (3, 3) & 39. \begin{cases} \frac{x-y}{4} = 1 \\ \frac{x}{2} - \frac{y}{4} = 3 \end{cases} & (8, 4) \\
31. \begin{cases} \frac{x-y}{2} = 2 \\ \frac{x+2y}{3} = y + \frac{4}{3} \end{cases} & (2, -2) & 40. \begin{cases} \frac{x+1}{4} + \frac{y-1}{2} = 3 \\ \frac{y-x}{2} = \frac{x}{3} \end{cases} & (3, 5)
\end{array}$$

## 2. Sistemi lineari: Metodo del confronto

Parte Base 2. Risolvi i seguenti sistemi lineari utilizzando il metodo del confronto:

$$1. \begin{cases} y = 2x - 1 \\ y = x + 3 \end{cases} \quad (4, 7) \quad 4. \begin{cases} x = 2y \\ x = -y + 6 \end{cases} \quad (4, 2)$$

$$2. \begin{cases} x = 3y + 2 \\ x = y - 4 \end{cases} \quad (-7, -3) \quad 5. \begin{cases} y = \frac{1}{2}x + 1 \\ y = x - 1 \end{cases} \quad (4, 3)$$

$$3. \begin{cases} y = -x + 5 \\ y = 2x - 4 \end{cases} \quad (3, 2) \quad 6. \begin{cases} x = y + 1 \\ x = -2y + 7 \end{cases} \quad (3, 2)$$

- |     |   |          |     |  |          |
|-----|---|----------|-----|--|----------|
| 7.  | $\begin{cases} y = 3x - 2 \\ y = -x + 6 \end{cases}$                  | (2, 4)   | 22. | $\begin{cases} 3(x + y) = y + 5 \\ x - y = 2(x - 2) \end{cases}$           | (-3, 7)  |
| 8.  | $\begin{cases} x = 4y - 3 \\ x = 2y + 5 \end{cases}$                  | (13, 4)  | 23. | $\begin{cases} \frac{x-1}{2} = y \\ \frac{x+y}{7} = y - 1 \end{cases}$     | (5, 2)   |
| 9.  | $\begin{cases} y = -2x + 1 \\ y = x - 5 \end{cases}$                  | (2, -3)  | 24. | $\begin{cases} 2(x - y) = x + 4 \\ 3x + y = 2(x - y) - 1 \end{cases}$      | (2, -1)  |
| 10. | $\begin{cases} 2y = x + 4 \\ 2y = -x + 8 \end{cases}$                 | (2, 3)   | 25. | $\begin{cases} \frac{y-1}{3} = x - 2 \\ \frac{y+2}{2} = x \end{cases}$     | (3, 4)   |
| 11. | $\begin{cases} 3x = 2y + 1 \\ 3x = y + 5 \end{cases}$                 | (3, 4)   | 26. | $\begin{cases} 2x - y = x + 1 \\ 3(x - 2) = 2y \end{cases}$                | (4, 3)   |
| 12. | $\begin{cases} y = x - 2 \\ y = -3x + 10 \end{cases}$                 | (3, 1)   | 27. | $\begin{cases} y - 2x = 5 \\ 3(y - x) = y - x + 4 \end{cases}$             | (-3, -1) |
| 13. | $\begin{cases} x = 5y + 2 \\ x = 2y + 11 \end{cases}$                 | (17, 3)  | 28. | $\begin{cases} \frac{x+y}{3} = x + 1 \\ \frac{y-x}{2} = x + 1 \end{cases}$ | (1, 5)   |
| 14. | $\begin{cases} y = 4x - 7 \\ y = 2x - 1 \end{cases}$                  | (3, 5)   | 29. | $\begin{cases} x - 2y = y \\ 2(x - y) = x + 2 \end{cases}$                 | (6, 2)   |
| 15. | $\begin{cases} x = 3 - 2y \\ x = 4y - 9 \end{cases}$                  | (-1, 2)  | 30. | $\begin{cases} 3x - y = -1 \\ 2(x - y) = x - y - 1 \end{cases}$            | (0, 1)   |
| 16. | $\begin{cases} 2x = y + 3 \\ 2x = 3y - 7 \end{cases}$                 | (4, 5)   | 31. | $\begin{cases} \frac{y}{2} = x - 1 \\ \frac{x+y}{4} = x - 1 \end{cases}$   | (2, 2)   |
| 17. | $\begin{cases} y = 5x + 2 \\ y = 3x + 6 \end{cases}$                  | (2, 12)  | 32. | $\begin{cases} x + 2y = y - 1 \\ 2(x + y) = x + 1 \end{cases}$             | (-3, 2)  |
| 18. | $\begin{cases} x = 7y - 1 \\ x = 5y + 3 \end{cases}$                  | (13, 2)  | 33. | $\begin{cases} 3(x - y) = 2x + 2 \\ x + y = 2(y + 2) \end{cases}$          | (5, 1)   |
| 19. | $\begin{cases} 3y = 2x - 4 \\ 3y = x + 1 \end{cases}$                 | (5, 2)   | 34. | $\begin{cases} \frac{x-y}{5} = 1 \\ \frac{2x+y}{4} = 1 \end{cases}$        | (3, -2)  |
| 20. | $\begin{cases} y = -3x + 2 \\ y = -5x - 4 \end{cases}$                | (-3, 11) | 35. | $\begin{cases} x - y = 2y - 1 \\ 2(x - y) = x - y + 1 \end{cases}$         | (2, 1)   |
| 21. | $\begin{cases} y + 2x = 3(x - y) - 2 \\ 2(x - y) = x + 2 \end{cases}$ | (2, 0)   |     |  |          |

$$\begin{array}{ll}
36. \begin{cases} \frac{y+2}{2} = x - 4 \\ \frac{x+1}{2} = y \end{cases} & (7, 4) \\
37. \begin{cases} x + 2 = y - 5 \\ 2(x + y) = x + y + 3 \end{cases} & (-2, 5) \\
38. \begin{cases} y - 2x = 2x - 3 \\ y + x = -x - 3 \end{cases} & (0, -3)
\end{array}
\qquad
\begin{array}{ll}
39. \begin{cases} \frac{x-2}{3} = -y \\ \frac{x-y}{5} = 2 \end{cases} & (8, -2) \\
40. \begin{cases} y - x = 2x - 3 \\ 2y + 2x = y + x + 9 \end{cases} & (3, 6)
\end{array}$$

### 3. Sistemi lineari: Metodo di riduzione

**Parte Base 3.** Risolvi i seguenti sistemi lineari utilizzando il metodo di riduzione:

$$\begin{array}{ll}
1. \begin{cases} x + y = 5 \\ x - y = 1 \end{cases} & (3, 2) \\
2. \begin{cases} 2x + y = 7 \\ 2x - y = 1 \end{cases} & (2, 3) \\
3. \begin{cases} 3x + 2y = 12 \\ x - 2y = 4 \end{cases} & (4, 0) \\
4. \begin{cases} x + 3y = 10 \\ -x + 2y = 5 \end{cases} & (1, 3) \\
5. \begin{cases} 4x - y = 6 \\ 2x + y = 6 \end{cases} & (2, 2) \\
6. \begin{cases} 5x + 3y = 13 \\ 2x - 3y = 1 \end{cases} & (2, 1) \\
7. \begin{cases} -3x + 4y = 11 \\ 3x + y = -1 \end{cases} & (-1, 2) \\
8. \begin{cases} 2x + 5y = 19 \\ -2x + y = -1 \end{cases} & (2, 3) \\
9. \begin{cases} x + 2y = 7 \\ 3x - y = 7 \end{cases} & (3, 2) \\
10. \begin{cases} 4x - 3y = 5 \\ 2x + y = 5 \end{cases} & (2, 1)
\end{array}
\qquad
\begin{array}{ll}
11. \begin{cases} 3x + 4y = 18 \\ 2x - 4y = -8 \end{cases} & (2, 3) \\
12. \begin{cases} 5x - 2y = 4 \\ 3x + 2y = 12 \end{cases} & (2, 3) \\
13. \begin{cases} x - y = -1 \\ 3x + y = 13 \end{cases} & (3, 4) \\
14. \begin{cases} 6x + 5y = 27 \\ 2x - 5y = -11 \end{cases} & (2, 3) \\
15. \begin{cases} -x + 3y = 8 \\ x + y = 4 \end{cases} & (1, 3) \\
16. \begin{cases} 4x + 3y = 17 \\ -4x + y = -5 \end{cases} & (2, 3) \\
17. \begin{cases} 7x - 2y = 12 \\ x + 2y = 4 \end{cases} & (2, 1) \\
18. \begin{cases} 2x - 7y = -12 \\ -2x + 3y = 4 \end{cases} & (1, 2) \\
19. \begin{cases} 3x + 5y = 21 \\ -3x + 2y = 0 \end{cases} & (2, 3) \\
20. \begin{cases} x + y = 0 \\ 2x - y = 6 \end{cases} & (2, -2)
\end{array}$$

$$\begin{array}{ll}
21. \begin{cases} \frac{x-1}{2} + y = 3 \\ 2(x-y) = x-1 \end{cases} & (3, 2) \\
22. \begin{cases} 3(x-1) + y = 10 \\ 2x - (y-x) = 11 \end{cases} & (4, 1) \\
23. \begin{cases} \frac{x+y}{2} - 2y = \frac{5}{2} \\ 2(x+y) + y = 1 \end{cases} & (2, -1) \\
24. \begin{cases} 2(2x+y) - y = 8 \\ x - \frac{3}{4}y = -2 \end{cases} & (1, 4) \\
25. \begin{cases} 2(x-y) = 3y \\ \frac{x}{5} + y = 3 \end{cases} & (5, 2) \\
26. \begin{cases} 3(x+1) + 2y = 3 \\ 3(y-x) + y = 18 \end{cases} & (-2, 3) \\
27. \begin{cases} 5(x+1) + 2(y-1) = 13 \\ 3x = 2(y-5) \end{cases} & (0, 5) \\
28. \begin{cases} \frac{x+1}{4} + y = -1 \\ x - 2(2y+1) = 9 \end{cases} & (3, -2) \\
29. \begin{cases} 7(x+1) - 3(y+1) = 0 \\ 2x + 3(y+2) = 1 \end{cases} & (-1, -1) \\
30. \begin{cases} \frac{x}{2} + 3y = 2 \\ x - 3(2y-1) = 7 \end{cases} & (4, 0) \\
31. \begin{cases} 4(2x-y) - y = 6 \\ 3(x-2) + 5y = 10 \end{cases} & (2, 2) \\
32. \begin{cases} x + 3(2y+1) = 7-y \\ -(x+1) + 2y = 4 \end{cases} & (-3, 1) \\
33. \begin{cases} 4(x-1) + 3y = -9 \\ 2x - 3(y+2) = 5 \end{cases} & (1, -3) \\
34. \begin{cases} \frac{2x-y}{5} = 1 \\ \frac{x+y}{2} = 5 \end{cases} & (5, 5) \\
35. \begin{cases} x - 5(y+1) = 1 \\ -(x+2) + 2y = -2 \end{cases} & (-4, -2) \\
36. \begin{cases} 3(x-2) - 4y = 8 \\ x + 4(y-1) = 6 \end{cases} & (6, 1) \\
37. \begin{cases} 5(x-1) + y = 1 \\ 3(x+1) - y = 13 \end{cases} & (2, -4) \\
38. \begin{cases} \frac{x}{2} - y = 2 \\ -(x-1) + 5y = -12 \end{cases} & (-2, -3) \\
39. \begin{cases} 2(x+y) + 7y = 33 \\ 4x - 3(3y-1) = -12 \end{cases} & (3, 3) \\
40. \begin{cases} 7(x+1) + 8y = 7 \\ 3x - 8(y-1) = 8 \end{cases} & (0, 0)
\end{array}$$

#### 4. Sistemi lineari: Metodo di Cramer

**Parte Base 4.** Risolvi i seguenti sistemi lineari utilizzando il metodo di Cramer:

$$\begin{array}{ll}
1. \begin{cases} x + y = 4 \\ x - y = 2 \end{cases} & (3, 1) \\
2. \begin{cases} 2x + y = 5 \\ 3x - y = 5 \end{cases} & (2, 1) \\
3. \begin{cases} x - 2y = -3 \\ 2x + y = 4 \end{cases} & (1, 2) \\
4. \begin{cases} 3x + 4y = 10 \\ 2x - y = 3 \end{cases} & (2, 1) \\
5. \begin{cases} 5x - 2y = 1 \\ x + 3y = 7 \end{cases} & (1, 2) \\
6. \begin{cases} x + 4y = 9 \\ 3x - 2y = 13 \end{cases} & (5, 1)
\end{array}$$

- |     |   |         |     |   |         |
|-----|---|---------|-----|---|---------|
| 7.  | $\begin{cases} 2x - 3y = -1 \\ 4x + y = 5 \end{cases}$    | (1, 1)  | 21. | $\begin{cases} \frac{x-y}{2} = 1 \\ x = 2 - \frac{y}{3} \end{cases}$        | (2, 0)  |
| 8.  | $\begin{cases} 3x - 5y = -1 \\ 2x + y = 8 \end{cases}$    | (3, 2)  | 22. | $\begin{cases} \frac{x+1}{3} = y \\ \frac{y-1}{2} = x - 4 \end{cases}$      | (4, 1)  |
| 9.  | $\begin{cases} 4x + 3y = 11 \\ 2x - 5y = -27 \end{cases}$ | (-1, 5) | 23. | $\begin{cases} \frac{x}{2} + \frac{y}{4} = 2 \\ x = y + 1 \end{cases}$      | (3, 2)  |
| 10. | $\begin{cases} x - 3y = -8 \\ 5x + 2y = 11 \end{cases}$   | (1, 3)  | 24. | $\begin{cases} \frac{2x-y}{3} = x - 2 \\ \frac{x+y}{2} = y + 1 \end{cases}$ | (4, 2)  |
| 11. | $\begin{cases} 2x + 3y = 12 \\ 3x - y = 7 \end{cases}$    | (3, 2)  | 25. | $\begin{cases} \frac{x+y}{3} = 2 \\ \frac{x-y}{2} = 2 \end{cases}$          | (5, 1)  |
| 12. | $\begin{cases} 5x + 4y = 22 \\ -x + 2y = 4 \end{cases}$   | (2, 3)  | 26. | $\begin{cases} \frac{2x+y}{7} = 1 \\ x = y - 1 \end{cases}$                 | (2, 3)  |
| 13. | $\begin{cases} 3x - 2y = 12 \\ 4x + 3y = 33 \end{cases}$  | (6, 3)  | 27. | $\begin{cases} \frac{x}{2} + y = 1 \\ \frac{x-y}{5} = 1 \end{cases}$        | (4, -1) |
| 14. | $\begin{cases} x + 5y = 17 \\ 2x - y = 1 \end{cases}$     | (2, 3)  | 28. | $\begin{cases} \frac{y+2}{3} = x \\ \frac{x+4y}{5} = 1 \end{cases}$         | (1, 1)  |
| 15. | $\begin{cases} 4x - y = 7 \\ 3x + 2y = 8 \end{cases}$     | (2, 1)  | 29. | $\begin{cases} \frac{x+y}{2} = 1 \\ \frac{y}{2} - x = 4 \end{cases}$        | (-2, 4) |
| 16. | $\begin{cases} 2x + 5y = 4 \\ 3x - 2y = 25 \end{cases}$   | (7, -2) | 30. | $\begin{cases} \frac{2x+y}{2} = 2 \\ \frac{x}{3} - y = 3 \end{cases}$       | (3, -2) |
| 17. | $\begin{cases} x - 4y = -14 \\ 2x + 3y = 5 \end{cases}$   | (-2, 3) | 31. | $\begin{cases} \frac{x+y}{5} = 1 \\ x = \frac{y-5}{3} \end{cases}$          | (0, 5)  |
| 18. | $\begin{cases} 5x - 3y = 19 \\ 4x + y = 22 \end{cases}$   | (5, 2)  | 32. | $\begin{cases} \frac{x}{2} - y = 1 \\ \frac{x+y}{4} = 2 \end{cases}$        | (6, 2)  |
| 19. | $\begin{cases} 3x + 7y = 27 \\ 2x - y = 1 \end{cases}$    | (2, 3)  | 33. | $\begin{cases} x = \frac{3-y}{4} \\ \frac{x-2y}{3} = 1 \end{cases}$         | (1, -1) |
| 20. | $\begin{cases} 6x - 5y = -3 \\ x + 2y = 8 \end{cases}$    | (2, 3)  | 34. | $\begin{cases} \frac{x-y}{2} = 1 \\ \frac{2x+y}{13} = 1 \end{cases}$        | (5, 3)  |

$$\begin{array}{ll}
35. \begin{cases} \frac{x-y}{4} = 0 \\ \frac{x+2y}{3} = -3 \end{cases} & (-3, -3) & 38. \begin{cases} \frac{2x+3y}{2} = 2 \\ \frac{x-y}{3} = -1 \end{cases} & (-1, 2) \\
36. \begin{cases} x = \frac{y-1}{3} \\ \frac{x+y}{3} = 3 \end{cases} & (2, 7) & 39. \begin{cases} \frac{5x-y}{2} = 7 \\ \frac{x}{2} = 2 - \frac{y}{2} \end{cases} & (3, 1) \\
37. \begin{cases} \frac{x+y}{2} = 2 \\ \frac{x-3y}{4} = 1 \end{cases} & (4, 0) & 40. \begin{cases} \frac{x}{2} + y = 2 \\ \frac{x-y}{5} = 2 \end{cases} & (8, -2)
\end{array}$$

## 5. Sistemi lineari: Esercizi di riepilogo

**Parte Base 5.** Risolvi i seguenti sistemi lineari prestando attenzione ai calcoli algebrici necessari per ridurli in forma normale:

$$\begin{array}{ll}
1. \begin{cases} \frac{x+4}{3} + \frac{3}{2}(x+y) = 0 \\ (\frac{1}{2}x - 1)(1+2x) + y = x^2 + 2 \\ (-\frac{10}{7}, \frac{6}{7}) \end{cases} & & 9. \begin{cases} 2x + 2y = 3 \\ 2(x+1)(x - \frac{1}{2}) = 2x^2 + (y - \frac{1}{2})^2 - y^2 \\ \text{imp.} \end{cases} \\
2. \begin{cases} \frac{2x-y}{3} - 2 = x \\ \frac{1}{6}x + \frac{y-1}{3} = -\frac{3}{2} \end{cases} & (-5, -1) & 10. \begin{cases} x = 6(y+1) \\ (x-1)^2 - 2(x+1)(x-1) = -3y - x^2 \\ (0, -1) \end{cases} \\
3. \begin{cases} \frac{2x-9y+2}{7} = \frac{3}{2}x \\ 16y + 5x = 9 \end{cases} & (-\frac{7}{13}, \frac{19}{26}) & 11. \begin{cases} x - 2y = \frac{1}{2} \\ (x-1+y)(x-1-y) = (x+y)(x-y) - 4y \\ \text{ind.} \end{cases} \\
4. \begin{cases} \frac{x+3}{2} - \frac{y-1}{6} = \frac{x+4}{3} \\ 2x + 3y + 19 = 0 \end{cases} & (-5, -3) & 12. \begin{cases} 6x + (y-2)(y+2) = (1-y)^2 + 4y \\ (2x - \frac{1}{2})^2 - x - y = 4x(x-1) \\ (\frac{11}{8}, \frac{13}{8}) \end{cases} \\
5. \begin{cases} 5(2x-5) = 6y \\ 5x - \frac{1}{2}(5+y) = \frac{1}{2}y \end{cases} & (-\frac{1}{2}, -5) & 13. \begin{cases} (x-y+2)^2 + 4x = (x-2)(x+2) - y(2x-y) \\ \frac{x-y}{3} = 2x + \frac{1}{2} \\ (-\frac{1}{2}, 1) \end{cases} \\
6. \begin{cases} 2x - 3y = 0 \\ (x + \frac{1}{2})^2 - (y + \frac{3}{2})^2 = (x-y)(x+y) - 2 \end{cases} & (0, 0) & 14. \begin{cases} (2x - 2y + 1)^2 + 8xy = 4(x^2 + y^2) + 3x \\ \frac{x-5}{4} - y = -\frac{1}{2} \\ \text{imp.} \end{cases} \\
7. \begin{cases} (3+y)(y-3) = \frac{x+y}{2} + y^2 - \frac{35}{4} \\ (x+4)^2 = 4y + (x - \frac{3}{2})(x+1) - \frac{11}{2} \end{cases} & (-2, \frac{3}{2}) & 15. \begin{cases} 5(x+9y) = 57 - 2y \\ (x+y)^2 - (x+1)^2 - (y+1)^2 = 2xy - 8 \end{cases} \\
8. \begin{cases} 5(x+9y) = 57 - 2y \\ (x+y)^2 - (x+1)^2 - (y+1)^2 = 2xy - 8 \end{cases} & (2, 1) & & \begin{cases} (x+1)^2 + y(y-4) = x^2 + y^2 - 3 \\ \frac{x+y}{2} = 2 \end{cases} \\
& & & (2, 2)
\end{array}$$

**Parte Avanzata 1.** Risolvi i seguenti sistemi lineari di livello avanzato, prestando attenzione allo sviluppo dei prodotti notevoli e alle semplificazioni:

16. 
$$\begin{cases} (x-1)^3 - x(x^2 - 3x) = y - 2x + 1 \\ \frac{x+y}{3} = \frac{x-y}{2} + 1 \end{cases} \quad \left(\frac{2}{3}, \frac{4}{3}\right)$$
17. 
$$\begin{cases} (x+1)^3 - (x-1)^3 = 6x^2 + y \\ \frac{2x-y}{4} + \frac{x+y}{3} = \frac{5}{6} \end{cases} \quad \left(\frac{4}{5}, 2\right)$$
18. 
$$\begin{cases} \frac{x+y}{3} = \frac{x-y}{2} \\ (x+2)(y-1) = xy + 8 \end{cases} \quad \left(-\frac{50}{3}, -\frac{10}{3}\right)$$
19. 
$$\begin{cases} (2x-y)^2 - 4x(x-y) = y^2 + x - 2y \\ \frac{x+1}{5} - \frac{y-2}{2} = 1 \end{cases} \quad (4, 2)$$
20. 
$$\begin{cases} \frac{x-1}{2} - \frac{y}{3} = \frac{1}{6} \\ (x+y)^2 - x(x+2y) = y^2 + x - 2 \end{cases} \quad (2, 1)$$
21. 
$$\begin{cases} 0, \bar{3}x + 0,5y = 2 \\ 1, \bar{6}x - y = 3 \end{cases} \quad (3, 2)$$
22. 
$$\begin{cases} (x-2)^3 - x(x-3)^2 = 3x - y \\ \frac{y-x}{2} + \frac{2x+y}{3} = 5 \end{cases} \quad (-10, 8)$$
23. 
$$\begin{cases} (x+y)(x-y) - x(x-2) = y(3-y) \\ \frac{1}{2}\left(x - \frac{1}{3}\right) + \frac{1}{3}\left(y + \frac{1}{2}\right) = x - y \end{cases} \quad (0, 0)$$
24. 
$$\begin{cases} \frac{2x-y}{3} - \frac{x+2y}{2} = 1 \\ x(x-2) + 16y = x^2 - 12 \end{cases} \quad \text{ind.}$$
25. 
$$\begin{cases} \frac{x+1}{2} + \frac{y-1}{3} = \frac{7}{6} \\ (x-y)^2 - (x+y)^2 + 4xy = 3x + 2y - 5 \end{cases} \quad \text{imp.}$$
26. 
$$\begin{cases} 0,5x - 0, \bar{6}y = -1 \\ \frac{x+y-1}{2} - \frac{x-y}{3} = \frac{7}{3} \end{cases} \quad (2, 3)$$
27. 
$$\begin{cases} 3 - [2x - (y-1)] = x + 2y \\ \frac{x+2}{3} - \frac{y-1}{2} = 2 \end{cases} \quad (1, -1)$$
28. 
$$\begin{cases} (x-1)(x+1)(x+2) - x^3 = y + 2x^2 \\ \frac{2x+y}{2} = \frac{x-y}{3} - 1 \end{cases} \quad (-4, 2)$$
29. 
$$\begin{cases} \frac{\frac{1}{2}x-1}{3} + \frac{y}{2} = \frac{1}{6} \\ \frac{x+\frac{y}{2}}{2} - \frac{x-y}{4} = 1 \end{cases} \quad (6, -1)$$
30. 
$$\begin{cases} (x+y+1)^2 - (x+y-1)^2 = 4x + 12 \\ \frac{x-1}{3} + \frac{y+2}{5} = 2 \end{cases} \quad (4, 3)$$

## 6. Sistemi lineari 3x3

**Parte Base 6.** Risolvi i seguenti sistemi lineari di tre equazioni in tre incognite:

1. 
$$\begin{cases} x + y + z = 6 \\ 2x - y + z = 3 \\ x + 2y - z = 2 \end{cases} \quad (1, 2, 3)$$
2. 
$$\begin{cases} x - y + z = 2 \\ x + y + z = 4 \\ 2x + y - z = 1 \end{cases} \quad (1, 1, 2)$$

$$3. \begin{cases} 2x + y - z = 5 \\ x - y + 2z = 3 \\ 3x + 2y + z = 14 \end{cases} \quad (2, 3, 2) \quad 12. \begin{cases} x + y + z = 1 \\ 2x - y + z = -2 \\ x - 2y - z = -4 \end{cases} \quad (-1, 1, 1)$$

$$4. \begin{cases} x + y - z = 0 \\ 2x - y + 3z = 9 \\ x + 3y + 2z = 13 \end{cases} \quad (1, 2, 3) \quad 13. \begin{cases} x + y + z = 1 \\ x - 2y + z = 4 \\ 2x + y - z = 0 \end{cases} \quad (1, -1, 1)$$

$$5. \begin{cases} x + 2y + z = 8 \\ -x + y + 2z = 7 \\ 2x - y - z = -3 \end{cases} \quad (1, 2, 3) \quad 14. \begin{cases} x + y + z = 4 \\ 2x - y - z = 2 \\ x + 2y + z = 3 \end{cases} \quad (2, -1, 3)$$

$$6. \begin{cases} x + y + z = 3 \\ 2x - y + z = 2 \\ 3x + y - 2z = 2 \end{cases} \quad (1, 1, 1) \quad 15. \begin{cases} x + y + z = 2 \\ x - y + 2z = -3 \\ 2x + y - z = -2 \end{cases} \quad (-2, 3, 1)$$

$$7. \begin{cases} x + y + z = 4 \\ x - y + 2z = 3 \\ 2x + y - z = 4 \end{cases} \quad (2, 1, 1) \quad 16. \begin{cases} x + y + z = 3 \\ 2x - y + z = 10 \\ x + 2y - z = -3 \end{cases} \quad (3, -2, 2)$$

$$8. \begin{cases} x + y + z = 4 \\ 2x - y + z = 1 \\ x + 2y - z = 4 \end{cases} \quad (1, 2, 1) \quad 17. \begin{cases} x + y + z = 3 \\ 2x - y + z = -1 \\ x + y - 2z = 0 \end{cases} \quad (0, 2, 1)$$

$$9. \begin{cases} x + y + z = 5 \\ x - y + 3z = 3 \\ 2x + y - z = 5 \end{cases} \quad (2, 2, 1) \quad 18. \begin{cases} x + y + z = 3 \\ 2x + y - z = 0 \\ x - 2y + z = 3 \end{cases} \quad (1, 0, 2)$$

$$10. \begin{cases} x + y + z = 5 \\ x - y + z = 3 \\ 2x + y - z = 6 \end{cases} \quad (3, 1, 1) \quad 19. \begin{cases} x + y + z = 3 \\ x - 2y + z = 0 \\ 3x + y - z = 7 \end{cases} \quad (2, 1, 0)$$

$$11. \begin{cases} x + y + z = 5 \\ 2x - y + z = 0 \\ x + y - z = 3 \end{cases} \quad (1, 3, 1) \quad 20. \begin{cases} x + y + z = -1 \\ 2x - y + z = 2 \\ x - 2y - z = 4 \end{cases} \quad (1, -1, -1)$$

## 7. Sistemi lineari fratti

**Parte Base 7.** Risolvi i seguenti sistemi lineari fratti, ricordando di determinare prima le Condizioni di Esistenza (C.E.):

- |     |  |          |     |  |                               |
|-----|--|----------|-----|--|-------------------------------|
| 1.  | $\begin{cases} \frac{x-1}{y} = 2 \\ \frac{x+1}{y-1} = 6 \end{cases}$               | (5, 2)   | 15. | $\begin{cases} \frac{x+2y}{x-y} = 2 \\ \frac{x-2}{y+1} = 1 \end{cases}$  | (4, 1)                        |
| 2.  | $\begin{cases} \frac{x+y}{x-y} = 7 \\ \frac{x}{y-1} = 2 \end{cases}$               | (4, 3)   | 16. | $\begin{cases} \frac{x-y}{x+y} = \frac{5}{11} \\ \frac{x-2}{y} = 2 \end{cases}$  | (8, 3)                        |
| 3.  | $\begin{cases} \frac{x+4}{y+1} = 1 \\ \frac{x-2}{y} = -4 \end{cases}$              | (-2, 1)  | 17. | $\begin{cases} \frac{x+y}{x-y} = -\frac{5}{3} \\ \frac{x-1}{y+2} = 1 \end{cases}$  | (-1, -4)                      |
| 4.  | $\begin{cases} \frac{x-y}{x+y} = -3 \\ \frac{2x+1}{y+1} = -3 \end{cases}$          | (1, -2)  | 18. | $\begin{cases} \frac{2x+y}{y-x} = \frac{13}{4} \\ \frac{x+1}{y-3} = 1 \end{cases}$   | (3, 7)                        |
| 5.  | $\begin{cases} \frac{x-1}{y-3} = 1 \\ \frac{x+y}{x} = \frac{8}{3} \end{cases}$     | (3, 5)   | 19. | $\begin{cases} \frac{x-y}{x+y} = 4 \\ \frac{x-3}{y+1} = -1 \end{cases}$  | (5, -3)                       |
| 6.  | $\begin{cases} \frac{x+2y}{x-y} = \frac{5}{2} \\ \frac{x}{y+1} = 2 \end{cases}$    | (6, 2)   | 20. | $\begin{cases} \frac{x+y}{x-y} = \frac{1}{2} \\ \frac{x+2}{y-4} = 2 \end{cases}$   | (-6, 2)                       |
| 7.  | $\begin{cases} \frac{x+y}{x} = -\frac{1}{2} \\ \frac{y-1}{x-1} = -4 \end{cases}$   | (2, -3)  | 21. | $\begin{cases} \frac{y}{y-5} - \frac{x}{x+1} = \frac{2x-3y+7}{xy-5x+y-5} \\ \frac{x-2}{3} + \frac{y}{2} = \frac{1}{6} \end{cases}$                   | (1, 1)                        |
| 8.  | $\begin{cases} \frac{x-y}{y} = 3 \\ \frac{x+2}{y-1} = 1 \end{cases}$               | (-4, -1) | 22. | $\begin{cases} \frac{x+y}{x-1} + \frac{2-x}{x+1} = \frac{x(y-3)}{x^2-1} \\ 3y + \frac{16x-3}{y+2} = \frac{1}{2}x - \frac{y(x-6y)}{2y+4} \end{cases}$ | $(\frac{1}{3}, -\frac{1}{3})$ |
| 9.  | $\begin{cases} \frac{2x-y}{x+y} = \frac{7}{8} \\ \frac{x-2}{y} = 1 \end{cases}$    | (5, 3)   | 23. | $\begin{cases} \frac{3}{x+y} - \frac{2}{x-y} = \frac{5}{x^2-y^2} \\ \frac{x+2}{4} - \frac{y-1}{3} = \frac{15}{12} \end{cases}$                       | (5, 0)                        |
| 10. | $\begin{cases} \frac{x+y}{y-x} = \frac{5}{3} \\ \frac{x+1}{y-2} = 1 \end{cases}$   | (1, 4)   | 24. | $\begin{cases} \frac{x+1}{y-2} + \frac{x-1}{y+2} = \frac{2x(y+1)}{y^2-4} \\ 2x - 3y = 5 \end{cases}$   | $(2, -\frac{1}{3})$           |
| 11. | $\begin{cases} \frac{x+y}{x-y} = \frac{5}{9} \\ \frac{y+4}{x-5} = 1 \end{cases}$   | (7, -2)  | 25. | $\begin{cases} (x+y+1)^2 - (x+y)^2 = 5 \\ \frac{2x-y}{x+1} = 1 \end{cases}$  | $(\frac{3}{2}, \frac{1}{2})$  |
| 12. | $\begin{cases} \frac{2x+y}{x+y} = -2 \\ \frac{x+4}{y-2} = \frac{1}{2} \end{cases}$ | (-3, 4)  | 26. | $\begin{cases} \frac{x}{x-1} + \frac{1}{1-x} = 2 \\ y = x + 1 \end{cases}$   | imp.                          |
| 13. | $\begin{cases} \frac{3x-y}{x+y} = \frac{1}{7} \\ \frac{y-x}{x+1} = 1 \end{cases}$  | (2, 5)   | 27. | $\begin{cases} \frac{x^2-y^2}{x-y} = 5 \\ (x+1)^2 - x^2 = y + 2 \end{cases}$   | (2, 3)                        |
| 14. | $\begin{cases} \frac{x-y}{x+y} = \frac{3}{7} \\ \frac{x+3}{y+1} = 2 \end{cases}$   | (-5, -2) |     |  |                               |

$$\begin{array}{ll}
28. \begin{cases} \frac{3}{x+y} - \frac{2}{x-y} = \frac{5}{x^2-y^2} \\ \frac{x+2}{4} - \frac{y-1}{3} = \frac{15}{12} \end{cases} & (5, 0) & 33. \begin{cases} \frac{x+y}{3} - \frac{x-y}{2} = \frac{1}{6} \\ (x+1)^2 - x(x+2) = y \end{cases} & (4, 1) \\
29. \begin{cases} \frac{x+1}{y-2} + \frac{x-1}{y+2} = \frac{2x(y+1)}{y^2-4} \\ 2x - 3y = 5 \end{cases} & (2, -\frac{1}{3}) & 34. \begin{cases} \frac{2}{x} + \frac{3}{y} = \frac{12}{xy} \\ 3x - 2y = 0 \end{cases} & (2, 3) \\
30. \begin{cases} (x+y+1)^2 - (x+y)^2 = 5 \\ \frac{2x-y}{x+1} = 1 \\ (\frac{3}{2}, \frac{1}{2}) \end{cases} & & 35. \begin{cases} \frac{x^2-y^2}{x+y} = 1 \\ \frac{2x+y}{x+1} = 2 \end{cases} & (3, 2) \\
31. \begin{cases} \frac{x}{x-1} + \frac{1}{1-x} = 2 \\ y = x + 1 \end{cases} & \text{imp.} & 36. \begin{cases} \frac{x+3}{y-1} - \frac{x}{y+1} = \frac{2x+9}{y^2-1} \\ x - 2y = 1 \end{cases} & (5, 2) \\
32. \begin{cases} \frac{x^2-y^2}{x-y} = 5 \\ (x+1)^2 - x^2 = y + 2 \end{cases} & (2, 3) & 37. \begin{cases} \frac{y+1}{x} = \frac{y-1}{x-2} \\ \frac{(x+y)^2 - (x-y)^2}{4} = 2 \end{cases} & (2, 1)
\end{array}$$

## 8. Natura del sistema (Determinati, Indeterminati, Impossibili)

**Parte Base 8.** Senza risolvere i sistemi, stabilisci se ciascuno di essi è determinato, indeterminato o impossibile confrontando i rapporti tra i coefficienti:

$$\begin{array}{ll}
1. \begin{cases} x + y = 2 \\ x + y = 5 \end{cases} & (\text{imp.}) & 8. \begin{cases} 5x - y = 2 \\ 5x - y = 3 \end{cases} & (\text{imp.}) \\
2. \begin{cases} 2x - y = 3 \\ 4x - 2y = 6 \end{cases} & (\text{indet.}) & 9. \begin{cases} x + y = 10 \\ x - y = 2 \end{cases} & (\text{det.}) \\
3. \begin{cases} x + 2y = 4 \\ 2x + y = 4 \end{cases} & (\text{det.}) & 10. \begin{cases} 4x - 2y = 8 \\ 2x - y = 4 \end{cases} & (\text{indet.}) \\
4. \begin{cases} 3x + y = 1 \\ 6x + 2y = 2 \end{cases} & (\text{indet.}) & 11. \begin{cases} 3x - 3y = 9 \\ x - y = 3 \end{cases} & (\text{indet.}) \\
5. \begin{cases} x - y = 4 \\ x - y = -4 \end{cases} & (\text{imp.}) & 12. \begin{cases} \frac{1}{2}x + y = 1 \\ x + 2y = 3 \end{cases} & (\text{imp.}) \\
6. \begin{cases} 2x + 3y = 6 \\ 4x + 6y = 12 \end{cases} & (\text{indet.}) & 13. \begin{cases} x = y + 1 \\ x - y = 1 \end{cases} & (\text{indet.}) \\
7. \begin{cases} x - 2y = 0 \\ 2x - 4y = 0 \end{cases} & (\text{indet.}) & 14. \begin{cases} 2x = 4y \\ x = 2y \end{cases} & (\text{indet.})
\end{array}$$

$$\begin{array}{ll}
15. \begin{cases} x + y = 0 \\ x + y = 1 \end{cases} & \text{(imp.)} & 18. \begin{cases} x + y = 5 \\ 2x + y = 10 \end{cases} & \text{(det.)} \\
16. \begin{cases} 2x - y = 5 \\ y = 2x + 5 \end{cases} & \text{(imp.)} & 19. \begin{cases} 4x + 6y = 10 \\ 2x + 3y = 5 \end{cases} & \text{(indet.)} \\
17. \begin{cases} 3x = y \\ y = 3x \end{cases} & \text{(indet.)} & 20. \begin{cases} x - 4y = 2 \\ 2x - 8y = 5 \end{cases} & \text{(imp.)}
\end{array}$$

## 9. Sistemi parametrici (k di primo grado)

**Parte Base 9.** Discuti la natura dei seguenti sistemi lineari al variare del parametro  $k \in \mathbb{R}$ :

$$\begin{array}{ll}
1. \begin{cases} kx + 3y = 4 \\ 2x - y = 1 \\ k = -6 \text{ imp)} \end{cases} & (k \neq -6 \text{ det;} \\
2. \begin{cases} 4x + ky = 2 \\ 2x + 3y = 1 \\ \text{indet)} \end{cases} & (k \neq 6 \text{ det;} k = 6 \\
3. \begin{cases} x + (k - 1)y = 3 \\ 2x + 4y = k \\ k = 3 \text{ imp)} \end{cases} & (k \neq 3 \text{ det;} \\
4. \begin{cases} 3x - 2y = 1 \\ (k + 2)x + 4y = -2 \\ k = -8 \text{ indet)} \end{cases} & (k \neq -8 \text{ det;} \\
5. \begin{cases} 5x + ky = 10 \\ x - 2y = 4 \\ k = -10 \text{ imp)} \end{cases} & (k \neq -10 \text{ det;} \\
6. \begin{cases} (2k - 1)x + 3y = 5 \\ x + y = 2 \\ k = 2 \text{ imp)} \end{cases} & (k \neq 2 \text{ det;} \\
7. \begin{cases} 2x - 5y = 3 \\ 4x + (k + 1)y = 6 \\ k = -11 \text{ indet)} \end{cases} & (k \neq -11 \text{ det;} \\
8. \begin{cases} x + 4y = k \\ 3x + (k + 5)y = 2 \\ k = 7 \text{ imp)} \end{cases} & (k \neq 7 \text{ det;} \\
9. \begin{cases} (k - 2)x + 2y = 4 \\ 3x - y = 1 \\ k = -4 \text{ imp)} \end{cases} & (k \neq -4 \text{ det;} \\
10. \begin{cases} 7x + 2y = 5 \\ kx - 4y = -10 \\ k = -14 \text{ indet)} \end{cases} & (k \neq -14 \text{ det;} \\
11. \begin{cases} x + (k + 3)y = 1 \\ 2x + y = 3 \\ k = -\frac{5}{2} \text{ imp)} \end{cases} & (k \neq -\frac{5}{2} \text{ det;} \\
12. \begin{cases} 4x - 3y = k \\ (k + 1)x - 6y = 2 \\ k = 7 \text{ imp)} \end{cases} & (k \neq 7 \text{ det;} \\
13. \begin{cases} x + 5y = 2 \\ (k - 1)x + 10y = 4 \\ k = 3 \text{ indet)} \end{cases} & (k \neq 3 \text{ det;} \\
14. \begin{cases} 3x + (2k - 1)y = 1 \\ x + 2y = 5 \\ k = \frac{7}{2} \text{ imp)} \end{cases} & (k \neq \frac{7}{2} \text{ det;} \\
15. \begin{cases} kx + 4y = 8 \\ 2x + y = 2 \\ \text{indet)} \end{cases} & (k \neq 8 \text{ det;} k = 8
\end{array}$$

$$\begin{array}{ll}
16. \begin{cases} x - 3y = 4 \\ 2x + (k - 2)y = 1 \\ k = -4 \text{ imp} \end{cases} & (k \neq -4 \text{ det}; \quad k = 1 \text{ indet}) \\
17. \begin{cases} (k + 4)x + y = 3 \\ 2x - 3y = -9 \\ k = -\frac{14}{3} \text{ indet} \end{cases} & (k \neq -\frac{14}{3} \text{ det}; \quad k = \frac{13}{5} \text{ imp}) \\
18. \begin{cases} 5x - (k + 1)y = 2 \\ 10x - 4y = 4 \end{cases} & (k \neq 1 \text{ det}; \quad k = 2 \text{ imp}) \\
19. \begin{cases} (3 - k)x + 2y = 7 \\ x + 5y = 1 \end{cases} & (k \neq \frac{13}{5} \text{ det}; \quad k = \frac{13}{5} \text{ imp}) \\
20. \begin{cases} 2x + (k - 5)y = k \\ 4x - 6y = -6 \end{cases} & (k \neq 2 \text{ det}; \quad k = 2 \text{ imp})
\end{array}$$

## 10. Sistemi lineari letterali con discussione

**Parte Base 10.** Risolvi e discuti i seguenti sistemi lineari al variare del parametro indicato:

$$1. \begin{cases} (a - 1)x + y = 1 \\ (a + 2)y - 2x - 1 = 0 \end{cases} \quad [a = -1: \text{ind.}; a = 0: \text{imp.}; a \neq -1 \wedge a \neq 0: (\frac{1}{a}, \frac{1}{a})]$$

$$2. \begin{cases} (k + 1)x + ky = -k \\ 2x + y = -2 \end{cases} \quad [k = 1: \text{imp.}; k \neq 1: (\frac{k}{1-k}, \frac{2}{k-1})]$$

$$3. \begin{cases} ax - y = 0 \\ (a - 2)x - y = a \end{cases} \quad [\forall a \in \mathbb{R}: (-\frac{a}{2}, -\frac{a^2}{2})]$$

$$4. \begin{cases} (a - 2)x - ay = 2 \\ ax - (1 + a)y = -1 \end{cases} \quad [a = -2: \text{imp.}; a \neq -2: (-\frac{3a+2}{a+2}, -\frac{3a-2}{a+2})]$$

$$5. \begin{cases} 4x - by = 20 \\ 2x + 5y = -b \end{cases} \quad [b = -10: \text{ind.}; b \neq -10: (\frac{10-b}{2}, -2)]$$

$$6. \begin{cases} -2kx + 3y = -k \\ (k + 1)x - 2y = 2 \end{cases} \quad [k = 3: \text{ind.}; k \neq 3: (2, k)]$$

$$7. \begin{cases} (k + 2)x - ky = 6 \\ 2x - y = k + 1 \end{cases} \quad [k = 2: \text{ind.}; k \neq 2: (k + 3, k + 5)]$$

$$8. \begin{cases} kx + 4y = 4 \\ 4x + ky = 8 - k \end{cases} \quad [k = 4: \text{ind.}; k = -4: \text{imp.}; k \neq \pm 4: (\frac{8}{k+4}, \frac{4-k}{k+4})]$$

$$9. \begin{cases} 2x - ay = a \\ 8x - 2y = 3 - 2a \end{cases} \quad [a = \frac{1}{2}: \text{ind.}; a \neq \frac{1}{2}: (-\frac{a}{4}, -\frac{3}{2})]$$

10.  $\begin{cases} bx = 2 \\ 3x + (b-1)y = 6 \end{cases} \quad [b = 0: \text{imp.}; b = 1: \text{ind.}; b \neq 0 \wedge b \neq 1: (\frac{2}{b}, \frac{6}{b})]$
11.  $\begin{cases} (a+1)x - ay = a \\ ax + ay = 0 \end{cases} \quad [a = 0: \text{ind.}; a = -\frac{1}{2}: \text{imp.}; a \neq 0 \wedge a \neq -\frac{1}{2}: (\frac{a}{2a+1}, -\frac{a}{2a+1})]$
12.  $\begin{cases} (a+5)x - 2y = a \\ (a+5)x + ay = -2 \end{cases} \quad [a = -2: \text{ind.}; a = -5: \text{imp.}; a \neq -2 \wedge a \neq -5: (\frac{a-2}{a+5}, -1)]$
13.  $\begin{cases} -6tx - 3ty = 3 \\ 4x + (1-t)y = 2 \end{cases} \quad [t = 0: \text{imp.}; t = -1: \text{ind.}; t \neq 0 \wedge t \neq -1: (\frac{1}{2t}, -\frac{2}{t})]$
14.  $\begin{cases} 3bx + (2+b)y = -3 \\ bx - by = -1 \end{cases} \quad [b = 0: \text{imp.}; b = -\frac{1}{2}: \text{ind.}; b \neq 0 \wedge b \neq -\frac{1}{2}: (-\frac{1}{b}, 0)]$
15.  $\begin{cases} -x + y = -1 \\ (a-2)x + y = a^2 - 2 \end{cases} \quad [a = 1: \text{ind.}; a \neq 1: (1+a, a)]$
16.  $\begin{cases} x - ay = 2a \\ ax - y = 2 \end{cases} \quad [a = \pm 1: \text{ind.}; a \neq \pm 1: (2, -2)]$
17.  $\begin{cases} kx + 2y = k \\ 2x + ky = -2 \end{cases} \quad [k = \pm 2: \text{imp.}; k \neq \pm 2: (\frac{k^2+4}{k^2-4}, -\frac{4k}{k^2-4})]$
18.  $\begin{cases} x - ay = a \\ 2x + y = 1 \end{cases} \quad [a = -\frac{1}{2}: \text{imp.}; a \neq -\frac{1}{2}: (\frac{2a}{2a+1}, \frac{1-2a}{2a+1})]$
19.  $\begin{cases} kx + (k-1)y = 1 \\ x + y = 1 \end{cases} \quad [\forall k \in \mathbb{R}: (2-k, k-1)]$
20.  $\begin{cases} 2x - ty = t \\ tx - 2y = 2 \end{cases} \quad [t = \pm 2: \text{ind.}; t \neq \pm 2: (0, -1)]$
21.  $\begin{cases} (a+1)x + 2y = a \\ 2x + (a+1)y = 0 \end{cases} \quad [a = 1 \vee a = -3: \text{imp.}; a \neq 1 \wedge a \neq -3: (\frac{a^2+a}{a^2+2a-3}, -\frac{2a}{a^2+2a-3})]$
22.  $\begin{cases} bx + 3y = b \\ 3x + by = 3 \end{cases} \quad [b = \pm 3: \text{ind.}; b \neq \pm 3: (1, 0)]$
23.  $\begin{cases} x + ky = k + 1 \\ kx + y = k + 1 \end{cases} \quad [k = \pm 1: \text{ind.}; k \neq \pm 1: (1, 1)]$

$$24. \begin{cases} (m-1)x + my = m+1 \\ mx + (m-1)y = m-1 \end{cases} \quad [m = \frac{1}{2}: \text{imp.}; m \neq \frac{1}{2}: (\frac{m-1}{1-2m}, \frac{1-3m}{1-2m})]$$

$$25. \begin{cases} ax + (a-1)y = a \\ (a+1)x + ay = 1 \end{cases} \quad [\forall a \in \mathbb{R}: (a^2 - a + 1, -a^2)]$$

## 11. Sistemi lineari con valore assoluto

**Parte Base 11.** Risolvi i seguenti sistemi lineari contenenti valori assoluti:

$$1. \begin{cases} |x| + y = 5 \\ 2x - y = 4 \end{cases} \quad (3, 2) \quad 11. \begin{cases} |x| = y \\ x + y = 6 \end{cases} \quad (3, 3)$$

$$2. \begin{cases} x + |y| = 4 \\ x - 2y = -2 \end{cases} \quad (2, 2) \quad 12. \begin{cases} |x - 2| = y + 1 \\ 2x - y = 4 \end{cases} \quad (\frac{5}{3}, -\frac{2}{3})$$

$$3. \begin{cases} |x - 1| + y = 3 \\ 2x + y = 4 \end{cases} \quad (\frac{2}{3}, \frac{8}{3}) \quad 13. \begin{cases} x + |y + 1| = 3 \\ 3x - y = 5 \end{cases} \quad (\frac{7}{4}, \frac{1}{4}) \vee (\frac{1}{2}, -\frac{7}{2})$$

$$4. \begin{cases} |2x| - y = 1 \\ x + y = 5 \end{cases} \quad (2, 3) \vee (-6, 11) \quad 14. \begin{cases} |2x - 1| = y \\ x - y = -1 \end{cases} \quad (2, 3) \vee (0, 1)$$

$$5. \begin{cases} x - |y - 2| = -1 \\ 3x + y = 7 \end{cases} \quad (1, 4) \vee (3, -2) \quad 15. \begin{cases} |x| + 2y = 4 \\ |x| - y = 1 \end{cases} \quad (2, 1) \vee (-2, 1)$$

$$6. \begin{cases} |x + y| = 4 \\ x - y = 2 \end{cases} \quad (3, 1) \vee (-1, -3) \quad 16. \begin{cases} x - |y| = 2 \\ 2x + |y| = 7 \end{cases} \quad (3, 1) \vee (3, -1)$$

$$7. \begin{cases} |x| + |y| = 5 \\ x - y = 1 \end{cases} \quad (3, 2) \vee (-2, -3) \quad 17. \begin{cases} |x - y| = 1 \\ |x + y| = 5 \end{cases} \quad (3, 2) \vee (2, 3) \vee (-2, -3) \vee (-3, -2)$$

$$8. \begin{cases} 2|x| + y = 7 \\ x + 3y = 6 \end{cases} \quad (3, 1) \vee (-\frac{15}{7}, \frac{19}{7}) \quad 18. \begin{cases} |x| + y = 4 \\ x - |y| = 0 \end{cases} \quad (2, 2)$$

$$9. \begin{cases} x - |2y| = 0 \\ 2x + y = 10 \end{cases} \quad (4, 2) \vee (\frac{20}{3}, -\frac{10}{3}) \quad 19. \begin{cases} |x| + y = 5 \\ 2x - |y| = 1 \end{cases} \quad (2, 3)$$

$$10. \begin{cases} |x - y| = 2 \\ x + 2y = 5 \end{cases} \quad (3, 1) \vee (\frac{1}{3}, \frac{7}{3}) \quad 20. \begin{cases} |x - 1| + |y - 2| = 2 \\ x - y = -1 \end{cases} \quad (2, 3) \vee (0, 1)$$