

# Scheda di Allenamento: Sistemi di Equazioni Fratte

## Sistemi lineari e riconducibili a lineari

### 1 Sistemi Fratti a due Incognite

**Parte Base 1.** Risolvi i seguenti sistemi, determinando preliminarmente le condizioni di esistenza (C.E.):

$$1. \begin{cases} \frac{x-y}{x+4} = 2 \\ \frac{x+5}{y+3} = -1 \end{cases} \quad [\textit{indeterminato}]$$

$$2. \begin{cases} \frac{2(1-2x)}{6-3y} = -1 \\ x + y = 3 \end{cases} \quad [(-1; 4)]$$

$$3. \begin{cases} \frac{1}{xy} = \frac{1}{x} - \frac{2}{y} \\ \frac{x+2}{x} = \frac{y+2}{y} + \frac{3}{xy} \end{cases} \quad [(\frac{1}{2}; 2)]$$

$$4. \begin{cases} \frac{14}{x} - \frac{10}{y} = \frac{13}{2x} + \frac{25}{2xy} \\ y - 3x = 0 \end{cases} \quad [(1; 3)]$$

$$5. \begin{cases} \frac{x+y}{x-y} = 3 \\ 2x - 3y = 5 \end{cases} \quad [(4; 1)]$$

$$6. \begin{cases} \frac{3x-y}{x+2} = 2 \\ \frac{y-1}{x-y} = -3 \end{cases} \quad [(3; 5)]$$

$$7. \begin{cases} \frac{x+2y}{y-x} = -1 \\ 3x + y = 2 \end{cases} \quad [(\frac{2}{3}; 0)]$$

$$8. \begin{cases} \frac{2}{x} - \frac{3}{y} = \frac{2y-3x}{xy} \\ \frac{x+y}{x} = 2 \end{cases} \quad [\textit{indeterminato, con } x \neq 0, y \neq 0]$$

$$9. \begin{cases} \frac{x-1}{y+1} = \frac{2}{3} \\ \frac{y-x}{x+2} = \frac{1}{4} \end{cases} \quad [(12; \frac{31}{2})]$$

$$10. \begin{cases} \frac{x(y+2)}{y-2} - x = \frac{2x+1}{y-2} \\ x + y = 5 \end{cases} \quad [(\frac{1}{2}; \frac{9}{2})]$$

$$11. \begin{cases} \frac{2}{x-1} + \frac{1}{y+2} = 1 \\ \frac{4}{x-1} - \frac{3}{y+2} = 7 \end{cases} \quad [S = \{(2, -3)\}]$$

12.  $\begin{cases} \frac{x^2+y}{x} = x + 3 \\ y - x = 2 \end{cases}$   $[S = \{(1, 3)\}]$
13.  $\begin{cases} \frac{x-y}{x+y} = \frac{1}{2} \\ \frac{x+1}{y-2} = 1 \end{cases}$   $[S = \{(9, 3)\}]$
14.  $\begin{cases} \frac{1}{x} - \frac{1}{y} = \frac{y-x}{xy} \\ x + y = 4 \end{cases}$   $[indeterminato, y = 4 - x \text{ con } x \neq 0 \wedge x \neq 4]$
15.  $\begin{cases} \frac{x+y}{x-y} = 2 \\ \frac{2x+2y}{x-y} = 5 \end{cases}$   $[S = \emptyset]$
16.  $\begin{cases} \frac{x-y}{x+1} = 1 \\ \frac{x+y}{y-2} = 2 \end{cases}$   $[S = (-5, -1)]$
17.  $\begin{cases} \frac{2x-y}{x+y} = \frac{1}{2} \\ \frac{x+2}{y-1} = 1 \end{cases}$   $[S = \emptyset]$
18.  $\begin{cases} \frac{3}{x} = \frac{2}{y} \\ x + y = 5 \end{cases}$   $[S = (3, 2)]$
19.  $\begin{cases} \frac{2x-y+1}{x-y} = 2 \\ x - 3y = 1 \end{cases}$   $[S = (-2, -1)]$
20.  $\begin{cases} \frac{1}{x-y} = \frac{2}{x+y} \\ \frac{x+2}{y} = 5 \end{cases}$   $[S = (3, 1)]$
21.  $\begin{cases} \frac{2y}{x-1} = 3 \\ 4x - y = 4 \end{cases}$   $[impossibile]$
22.  $\begin{cases} \frac{3}{2x+y} = -1 \\ x + y = 2 \end{cases}$   $[(-5, 7)]$
23.  $\begin{cases} 4x - 2y - 14 = 0 \\ \frac{2}{x} = \frac{3+y}{x^2-2x} \end{cases}$   $[indet., \text{ con } x \neq 0 \wedge x \neq 2]$
24.  $\begin{cases} \frac{x-2}{y} = 1 \\ \frac{y-4}{x} = -2 \end{cases}$   $[impossibile]$
25.  $\begin{cases} \frac{2}{x} - \frac{1}{x-8} = \frac{y}{2x^2-16x} \\ \frac{y-1}{x-2} + \frac{9}{10} = 0 \end{cases}$   $[(12, -8)]$

**Parte Avanzata 1.** *Semplifica le equazioni e risolvi i sistemi:*

$$26. \begin{cases} \frac{y(2x+1)}{x-1} - 3y = \frac{x(2-y)}{x-1} \\ 2y(1 - \frac{x}{2}) = -x(y - \frac{4}{x}) \end{cases} \quad [(4; 2)]$$

$$27. \begin{cases} \frac{x-2}{y+3} = \frac{x+1}{y-1} \\ 2x - y = 1 \end{cases} \quad [(-2; -5)]$$

$$28. \begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{x+y}{2} \\ x - y = 0 \end{cases} \quad [impossibile]$$

$$29. \begin{cases} \frac{3}{x+y} = \frac{2}{x-y} \\ 5x - y = 24 \end{cases} \quad [(5; 1)]$$

$$30. \begin{cases} \frac{2x-1}{y+2} = 1 \\ \frac{y+4}{x+1} = 2 \end{cases} \quad [(1; 1)]$$

$$31. \begin{cases} \frac{x-1}{y+1} = \frac{x}{y} \\ (x-1)^2 + y = x^2 + 2 \end{cases} \quad [(-\frac{1}{3}, \frac{1}{3})]$$

$$32. \begin{cases} \frac{x-y}{x^2-1} = \frac{1}{x+1} + \frac{2}{x-1} \\ x - y = 5 \end{cases} \quad [(\frac{4}{3}, -\frac{11}{3})]$$

$$33. \begin{cases} \frac{1}{x} + \frac{1}{x-y} = \frac{2y-x}{x^2-xy} \\ x + y = 1 \end{cases} \quad [\emptyset]$$

$$34. \begin{cases} \frac{2}{x+y} - \frac{1}{y-x} = \frac{x}{x^2-y^2} \\ 0, 2x + 0, 1y = 0, 5 \end{cases} \quad [indeterminato con x \neq \pm y]$$

$$35. \begin{cases} \frac{1}{x-y} = \frac{1}{x+y} - \frac{x+y-1}{x^2-y^2} \\ (x-1)^2 + y = (x-2)^2 \end{cases} \quad [(\frac{8}{5}, -\frac{1}{5})]$$

$$36. \begin{cases} \frac{x-3}{x-1} + \frac{x^2-xy+2}{x^2-x} = \frac{y}{x} \\ \frac{(x-1)^2-(x+2)^2}{y} = -1 \end{cases} \quad [S = \{(-\frac{1}{3}, 1)\}]$$

$$37. \begin{cases} \frac{x-1}{x} - \frac{y-2}{y} = \frac{1}{xy} \\ \frac{1}{9}x + \frac{1}{6}y = 1 \end{cases} \quad [S = \{(6, 2)\}]$$

$$38. \begin{cases} \frac{x}{x-2} - \frac{y-1}{y+1} = \frac{1}{2-x} \\ \frac{2}{x+1} + \frac{1}{y-2} = \frac{2+\frac{1}{2}}{xy-2x+y-2} \end{cases} \quad [S = \{(\frac{1}{2}, 0)\}]$$

$$39. \begin{cases} \frac{y-1}{x+1} + \frac{4}{2x+1} = \frac{2xy-2}{2x^2+3x+1} \\ \frac{x-2}{x} - \frac{y-2}{y} = \frac{1-y}{xy} \end{cases} \quad [S = \emptyset]$$

40. 
$$\begin{cases} \frac{3}{2x+2y} - \frac{4}{2x^2+2xy} = \frac{1}{2x} \\ \frac{1}{x} + \frac{1}{y+4} = \frac{2(x+y+4)}{xy+4x} \end{cases} \quad [S = \emptyset]$$
41. 
$$\begin{cases} \frac{3}{x+1} + \frac{2}{y-1} = 4 \\ \frac{1}{x+1} - \frac{4}{y-1} = -1 \end{cases} \quad [(0; 3)]$$
42. 
$$\begin{cases} \frac{x-\frac{1}{3}}{y} - 2 = \frac{3x-1}{3} - x \\ x + 3y = 4 \end{cases} \quad [(\frac{23}{14}, \frac{11}{14})]$$
43. 
$$\begin{cases} \frac{1}{x^2-y^2} + \frac{1}{x+y} = \frac{2}{x-y} \\ 2x - y = 3 \end{cases} \quad [(\frac{10}{7}, -\frac{1}{7})]$$
44. 
$$\begin{cases} \frac{3x-2}{2y+1} = \frac{3x+4}{2y-1} \\ \frac{2y+3}{x-2} = \frac{2y-1}{x} \end{cases} \quad [(\frac{4}{3}, -\frac{5}{6})]$$
45. 
$$\begin{cases} \frac{x+y}{x-y} - \frac{x-y}{x+y} = \frac{4}{x^2-y^2} \\ x + 2y = 3 \end{cases} \quad [(2; \frac{1}{2})]$$
46. 
$$\begin{cases} \frac{1}{x+y} + \frac{1}{x-y} = \frac{3}{4} \\ \frac{1}{x+y} - \frac{1}{x-y} = -\frac{1}{4} \end{cases} \quad [(3; 1)]$$
47. 
$$\begin{cases} \frac{2x-1}{y-2} = \frac{2x+3}{y+1} \\ 3x - 2y = 1 \end{cases} \quad [impossibile]$$
48. 
$$\begin{cases} \frac{3x-2}{2y-3} - \frac{3x-5}{2y+3} = \frac{54}{9-4y^2} \\ x + y = 1 \end{cases} \quad [(-2; 3)]$$
49. 
$$\begin{cases} \frac{x+2}{1+x} - \frac{x+1}{y} = 1 - \frac{x^2}{y+xy} \\ y - x = 1 \end{cases} \quad [(-2; -1)]$$
50. 
$$\begin{cases} \frac{3y-1}{x} = \frac{1+3x}{y} \\ x + y = 4 \end{cases} \quad [(2; 2)]$$
51. 
$$\begin{cases} \frac{4}{3} - 5 \cdot \frac{1}{y} = \frac{2-x}{y} \\ 2(x-1) = \frac{5}{3}y + 12 \end{cases} \quad [impossibile]$$
52. 
$$\begin{cases} \frac{x-y^2-1}{2y} = 2 - \frac{y}{2} \\ \frac{3x^2-6y+4}{3x-3} = x + \frac{8}{9} \end{cases} \quad [\{x = 7, y = \frac{3}{2}\}]$$
53. 
$$\begin{cases} \frac{1}{3} \left( \frac{x-2y}{2} - 3 \right) = \frac{1}{2} \left( \frac{3x-4y}{2} - y \right) - \frac{9}{2} \\ \frac{3[x-\frac{1}{4}(2y-1)]}{x+1} = \frac{13}{4} \end{cases} \quad [\{x = 2, y = -2\}]$$

$$54. \begin{cases} \frac{4x-3}{6y-3} = 1 \\ \frac{6x-3y}{5x} = \frac{x-6y-3}{3x} + \frac{38}{15} \end{cases} \quad [\textit{impossibile}]$$

$$55. \begin{cases} \frac{1}{x} - \frac{2}{y-2} = \frac{4}{xy-2x} \\ \frac{1}{x-1} = \frac{1}{y-2} \end{cases} \quad \left[ \begin{cases} x = -5 \\ y = -4 \end{cases} \right]$$

$$56. \begin{cases} \frac{x}{x^2-y^2} + \frac{1}{x+y} = \frac{5}{8} \\ \frac{2}{x-1} - \frac{y}{(y+1)^2} = \frac{3}{4} \end{cases} \quad [(3; 1)]$$

$$57. \begin{cases} \frac{x}{x-2} + \frac{y}{y-2} = \frac{4}{xy-2x-2y+4} \\ x + y = 6 \end{cases} \quad [\textit{impossibile}]$$

$$58. \begin{cases} \frac{3x+y}{x-y} = \frac{1}{3} \\ \frac{2}{x+2} + \frac{y}{y+2} = \frac{5}{2} \end{cases} \quad [(-1; 2)]$$

$$59. \begin{cases} \frac{x+y}{x-y} = \frac{1}{5} \\ \frac{2x}{3y+2} + \frac{4x+2}{y+1} = 7 \end{cases} \quad \left[ \left( \frac{1}{2}; -\frac{1}{3} \right) \right]$$

$$60. \begin{cases} \frac{x}{x+1} - \frac{y}{y-1} = \frac{2x-y}{xy-x+y-1} \\ \frac{x+2}{y+2} = \frac{1}{2} \end{cases} \quad [(0; 2)]$$

$$61. \begin{cases} \frac{2}{x-1} + \frac{3}{y+1} = \frac{5}{2} \\ \frac{2}{y+1} - \frac{1}{x-1} = \frac{1}{2} \end{cases} \quad [(3; 1)]$$

$$62. \begin{cases} \frac{x+y+2}{x-y} = 5 \\ \frac{2x-1}{y+1} = \frac{3}{2} \end{cases} \quad [(2; 1)]$$

$$63. \begin{cases} \frac{x^2-2xy+y^2}{x^2-y^2} = \frac{x-y}{x+y} \\ 3x - y = 4 \end{cases} \quad [\textit{indeterminato con } x \neq \pm y]$$

$$64. \begin{cases} \frac{1}{x-2} - \frac{1}{y+3} = \frac{5}{(x-2)(y+3)} \\ x + y = 8 \end{cases} \quad [(4; 4)]$$

$$65. \begin{cases} \frac{2}{x-1} + \frac{3}{y+2} = \frac{4}{(x-1)(y+2)} \\ x - y = 1 \end{cases} \quad [\textit{impossibile}]$$

## 2 Sistemi Fratti a Tre Incognite

**Parte Avanzata 2.** Risolvi i seguenti sistemi fratti, prestando attenzione alle condizioni di esistenza per ciascuna incognita:

$$66. \begin{cases} \frac{x+y}{z+2} = 3 \\ \frac{y-z}{x-1} = 2 \\ x + 2y + z = 3 \end{cases} \quad [(2; 1; -1)]$$

$$67. \begin{cases} \frac{x+y+z}{x-1} = \frac{6}{x-1} \\ 2x - y + z = 3 \\ \frac{x+2y-z}{y} = 1 \end{cases} \quad [impossibile]$$

$$68. \begin{cases} \frac{2x+y}{z+1} = 2 \\ \frac{y-z}{2x} = 2 \\ 4x + y - 2z = 3 \end{cases} \quad [(\frac{1}{2}; 3; 1)]$$

$$69. \begin{cases} \frac{x+z}{y+2} = 1 \\ \frac{3x-y}{z-2} = -3 \\ \frac{x+y+z}{x+4} = 1 \end{cases} \quad [(-2; 0; 4)]$$

$$70. \begin{cases} \frac{x+y}{z+6} = 1 \\ \frac{y-z}{x-2} = 1 \\ \frac{2x+3y+z}{x-2} = 1 \end{cases} \quad [(4; -1; -3)]$$

$$71. \begin{cases} \frac{2x+y}{z+2} = 4 \\ \frac{x-z}{y} = 1 \\ 3x + y + 2z = 3 \end{cases} \quad [(1; 2; -1)]$$

$$72. \begin{cases} \frac{x+y+z}{x-y} = \frac{3}{4} \\ \frac{2x-z}{y+2} = 5 \\ x + y - 2z = 0 \end{cases} \quad [(3; -1; 1)]$$

$$73. \begin{cases} \frac{y+z}{x+1} = 1 \\ \frac{2x+y}{z+3} = 3 \\ \frac{2x-2y}{z} = 3 \end{cases} \quad [(0; 3; -2)]$$

$$74. \begin{cases} \frac{x-y}{z} = \frac{3}{2} \\ \frac{x+z}{y+1} = 3 \\ \frac{y+z}{x-2} = \frac{3}{2} \end{cases} \quad [(4; 1; 2)]$$

$$75. \begin{cases} \frac{x+y}{z+2} = 2 \\ \frac{2x+2y-z}{z} = 3 \\ \frac{x-y}{z-1} = 1 \end{cases} \quad [impossibile]$$

### 3 Sistemi Riconducibili a Lineari (Cambi di Variabili)

**Parte Avanzata 3.** Risolvi i seguenti sistemi introducendo opportune variabili ausiliarie (ad esempio  $u = \frac{1}{x}$ ,  $v = \frac{1}{y}$ ):

$$76. \begin{cases} \frac{3}{x} - \frac{4}{y} = 7 \\ \frac{1}{x} + \frac{2}{y} = 1 \end{cases} \quad \left[ \left( \frac{5}{9}; -\frac{5}{2} \right) \right]$$

$$77. \begin{cases} \frac{1}{x} + \frac{2}{y} = 8 \\ \frac{3}{x} - \frac{1}{y} = 3 \end{cases} \quad \left[ \left( \frac{1}{2}; \frac{1}{3} \right) \right]$$

$$78. \begin{cases} \frac{1}{2x} + \frac{1}{y} = 1 \\ \frac{3}{x} - \frac{1}{y} = \frac{5}{2} \end{cases} \quad [(1; 2)]$$

$$79. \begin{cases} \frac{x+1}{x} - \frac{y+1}{y} + \frac{1}{2} = 0 \\ \frac{1}{x} + \frac{1}{y} = \frac{3}{2} \end{cases} \quad [(2; 1)]$$

$$80. \begin{cases} \frac{7}{2x+3y} + \frac{1}{x-2y} = -\frac{12}{19} \\ \frac{1}{2x+3y} - \frac{2}{x-2y} = \frac{39}{19} \end{cases} \quad [(5; 3)]$$

$$81. \begin{cases} \frac{2}{x} - \frac{1}{y} = 3 \\ \frac{1}{x} + \frac{2}{y} = 4 \end{cases} \quad \left[ \left( \frac{1}{2}; 1 \right) \right]$$

$$82. \begin{cases} \frac{1}{x} - \frac{3}{4y} + 2 = 0 \\ \frac{2}{3x} + \frac{3}{2y} = \frac{20}{3} \end{cases} \quad \left[ \left( 1; \frac{1}{4} \right) \right]$$

$$83. \begin{cases} \frac{3}{x+2} + \frac{2}{y-3} = 2 \\ \frac{6}{x+2} - \frac{4}{y-3} = 0 \end{cases} \quad [(1; 5)]$$

$$84. \begin{cases} \frac{x+1}{y} + \frac{2(y+2)}{x} = 6 \\ \frac{2(x+1)}{y} - \frac{2(y+2)}{x} = 3 \end{cases} \quad [(2; 1)]$$

$$85. \begin{cases} \frac{4}{3x-1} + \frac{3}{2y+1} = 3 \\ \frac{2}{3x-1} - \frac{6}{2y+1} = -1 \end{cases} \quad [(1; 1)]$$

$$86. \begin{cases} \frac{2(x-2)}{y+1} + \frac{7(y-1)}{x+2} = 3 \\ \frac{x-2}{y+1} - \frac{14(y-1)}{x+2} = -1 \end{cases} \quad [(5; 2)]$$

$$87. \begin{cases} \frac{4}{x+y} + \frac{3}{x-y} = \frac{5}{2} \\ \frac{8}{x+y} - \frac{3}{x-y} = \frac{1}{2} \end{cases} \quad [(3; 1)]$$

$$88. \begin{cases} \frac{x+2y}{x-1} - \frac{x-2y}{x+1} = \frac{4xy+4}{x^2-1} \\ \frac{y-1}{x-3} = \frac{y+1}{x} \end{cases} \quad \left[ \left( 2; \frac{1}{3} \right) \right]$$

$$89. \begin{cases} \frac{3}{x-y} - \frac{2}{x+y} = \frac{9}{x^2-y^2} \\ \frac{2x-3}{y+4} = \frac{2x-5}{y+2} \end{cases} \quad [(4; 1)]$$

$$90. \begin{cases} \frac{2x+y-1}{x-1} + \frac{3x-y+2}{x+1} = \frac{5x^2-4x+3}{x^2-1} \\ \frac{x}{y-1} - \frac{x-2}{y+1} = \frac{2x+6}{y^2-1} \end{cases} \quad [(-\frac{1}{2}; 4)]$$